THE RELATIONSHIP OF PASS-THROUGH OF EXCHANGE RATES TO PURCHASING POWER PARITY*

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Abstract

We develop and test two hypotheses about purchasing power parity (PPP) derived from the pricing behaviour of profit-maximising, exporting firms. The first is that changes in the price of traded goods relative to domestic substitutes will affect the PPP relation, due to the partial pass-through of exchange rates. The second is that PPP should hold on forward rather than spot exchange rates, due to hedging by firms, which implies that the interest rate differential should enter the PPP relation for spot rates. Using quarterly data for the United States, Canada, Germany, Japan and the United Kingdom, we find support for both these hypotheses, though the magnitude of the interest rate effect is very small.

1. Introduction

We develop and test a model of purchasing power parity (PPP) derived from the optimal pricing behaviour of exporting firms. Under imperfect competition, exporting firms will likely adjust their prices by less than the full change in the exchange rate. For example, as their currency appreciates, firms may lower their profit margins to absorb part of the exchange rate change, thereby passing through only part of the appreciation to the importer's price. This change in the price of traded goods relative to domestic substitutes, due to pass-through behaviour, should be taken into account when measuring the parity between prices in the exporting and importing countries. This is the first hypothesis that we shall investigate.

The second hypothesis we consider is that parity should hold between the prices in trading partners and their *forward rates* of foreign exchange, rather than their spot rates. From covered interest parity, the difference between the spot and forward rates equals the interest rate differential, so this second hypothesis implies that PPP equations of spot rates should include the interest rate differential as an explanatory variable. We will find considerable support for our first hypothesis, but less support for the second: while the interest rate differential is a significant variable in the PPP relation, we find that the magnitude of this effect is very small. One reason for this is that the interest rate differentials between most countries are stationary, or nearly so, so they cannot explain the nonstationary deviations from PPP. A significant portion of these deviations are, however, explained by the price of traded goods relative to domestic substitutes.

There is ample precedent in the literature for both the hypotheses that we test. The idea that partial pass-through of exchange rates may affect PPP is considered by Froot and Rogoff (1995), though they devote greater attention to a more conventional hypothesis: that deviations from PPP will arise due to the inclusion of nontraded goods in the wholesale or consumer price indexes. The implication of this hypothesis seems to be that we should correct the aggregate price indices, possibly by including the relative price of traded goods as another variable in the PPP relation. Thus, the correction implied by the mis-measurement of indices (due to nontraded goods) is quite similar to the correction we propose to account for pass-through behaviour, and in this sense the two hypotheses are similar. Nevertheless, we will argue that there are some subtle differences in the exact manner in which these two hypotheses should be tested (see section 3.2).

The idea that the forward rate determines the price and/or output for exporters is also not new, and is an example of the "separation theorem" discussed by Ethier (1973), Baron (1976a) and Eldor and Zilcha (1987). We will derive this result from a model of a risk-averse, exporting firm, that must set the prices for its products before the exchange rate is known. The firm may set prices in either its own currency, or the currency of the importing country, and will optimally engage in transactions in the forward market. In either case, we show in section 2 that the optimal price for the firm is determined by the forward rate, even if only partial covering is optimal.

The optimal pricing relation for the firm can be estimated as a pass-through equation between forward rates and product prices, or alternatively, inverted to obtain a PPP relation between the product prices and the forward rate, as described in section 3. In section 4 we estimate the latter as a cointegrating relation, using quarterly data for the United States, Canada, Germany, and the United Kingdom over 1974.1-1994.4. Applying the method of Johansen (1991), we find strong evidence of *multiple* cointegrating relations, so that not all variables need be included to obtain a stationary relation with the exchange rate. We find that the cointegrating vectors that include the relative traded goods price have significantly lower residuals, or smaller deviations from PPP, than the relations that only use wholesale prices. In contrast, interest rate differentials explain very little of the PPP deviations in most cases. Additional conclusions and directions for further research are discussed in section 5.

2. Theory

Numerous authors have examined the question of the currency in which exporting firms set their prices. Studies which have examined the optimal choice of invoicing strategy include those by Baron (1976b) and Giovannini (1988), with the result that the optimal choice is very sensitive to properties of the demand function. We shall consider invoicing in either the currency of the exporter or in the currency of the importing country.

2.1. Invoicing in the Importing Country's Currency

In this section we suppose that the firm sets its price in the importing country's currency. The model we shall use is similar to Feenstra (1989), except that the firm is assumed to be risk averse. Buying one unit of importer's currency requires s_t units of the exporter's currency on the spot market, where s_t is stochastic. Since the exporter must set its price p_t in period t-1 (before this spot rate is known) the revenues received in its own currency are uncertain. This uncertainty can be covered by selling in period t-1 the amount y_t of the importer's currency on the forward market, at the price of ${}_{t-1}f_t$. The firm will experience a profit (or loss) on these forward contracts of $y_t({}_{t-1}f_t - s_t)$, which will offset the “translation exposure” from converting sales revenue to its own currency. The demand for imports is given by $x_t=x(p_t,q_t,I_t)$, where q_t is the (scalar) price of domestic import-competing goods, and I_t is consumer income or expenditure.[1] We assume that the firm is engaged in Bertrand competition with other firms, so it treats q_t as exogenous.

The exporter maximises expected utility of profits in its own currency:

$$\max_{p_t, y_t} E_{t-1}\{U[(s_t p_t - c^*_t)x(p_t,q_t,I_t) + y_t({}_{t-1}f_t - s_t)]\}, \qquad (1)$$

where E_{t-1} denotes expected value using information available in period t-1; U is the firm's utility function; and c^*_t denotes marginal and average costs in the foreign currency. We will treat both costs and consumer income as stochastic, but independent of each other and of the spot rate. The firm will be forecasting the period t values of these variables using information available in t-1. The price q_t is chosen by firms in period t-1, using an analogous maximisation problem to (1). This price will be fully determined by information available in period t-1, so we will treat it as nonstochastic in (1).

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[1] Note that the prices of domestic goods q_t could be a vector, but for convenience we shall treat it as a scalar aggregate. In addition, we assume that this competing good is an imperfect substitute for the product of the exporting firm.

Let $\pi^*_t = (s_t p_t - c^*_t)x(p_t, q_t, I_t) + y_t({}_{t-1}f_t - s_t)$ denote profits in the exporting country's currency, inclusive of the gain or loss from the forward transaction. Then $U(\pi^*_t)$ may be approximated by a second order Taylor expansion about expected profits as:

$$U(\pi^*_t) \approx U(E_{t-1}\pi^*_t) + U'(E_{t-1}\pi^*_t)(\pi^*_t - E_{t-1}\pi^*_t)$$
$$+ \frac{1}{2}U''(E_{t-1}\pi^*_t)(\pi^*_t - E_{t-1}\pi^*_t)^2. \qquad (2)$$

Letting $e_t = E_{t-1}(s_t)$ denote the expected exchange rate, using information available in t-1, and substituting (2) into equation (1) we obtain:

$$\max_{p_t, y_t} U(E_{t-1}\pi^*_t) + \frac{1}{2}U''(E_{t-1}\pi^*_t)\mathrm{var}_{t-1}(\pi^*_t), \qquad (3)$$

where, $$E_{t-1}\pi^*_t = (e_t p_t - E_{t-1}c^*_t)E_{t-1}x(p_t, q_t, I_t) + y_t({}_{t-1}f_t - e_t), \qquad (4)$$

is the conditional mean of profits, and $\mathrm{var}_{t-1}(\pi^*_t) = \sigma^2_s(p_t E_{t-1}x_t - y_t)^2$, and $\sigma^2_s \equiv E_{t-1}(s_t - e_t)^2$ denote conditional variances of profits and the spot rate, respectively. Assuming that $U''(E_{t-1}\pi^*_t)$ is constant,[2] the first-order condition of (3) with respect to y_t is:

$$y_t = \frac{({}_{t-1}f_t - e_t)}{R_u \sigma^2_s} + p_t E_{t-1}x(p_t, q_t, I_t), \qquad (5)$$

where $R_u \equiv -[U''(E_{t-1}\pi^*_t)/U'(E_{t-1}\pi^*_t)]$ is the Arrow-Pratt measure of absolute risk-aversion. From (5), the optimal forward contract is decomposed into two terms: the first is a “speculative” purchase (or sale) that reflects the difference between the forward and expected future spot rate; while the second term is the sales revenue that the firm needs to convert to its own currency. If ${}_{t-1}f_t < e_t$, indicating that the exporter expects an appreciation of the importer's currency relative to the current forward rate, then the optimal speculative position is to buy forward contracts, so in that case the firm will not sell forward enough of the importer's currency to convert its total sales revenue. The speculative purchase or sale is also affected by

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[2] Thus, our analysis is exact for a quadratic utility function U.

the firm's attitude towards risk, as indicated by R_u. The relation between the forward and expected future spot rate is determined in general equilibrium (as in Hodrick, 1989, for example), and is related to the risk premium in the foreign exchange market. We will simply accept these rates as exogenous to the firm.

Before determining the optimal price p_t, it is useful to substitute (5) back into (4b) and rewrite the conditional variance of profits as:

$$\mathrm{var}_{t-1}(\pi_t^*) = \frac{({}_{t-1}f_t - e_t)^2}{R_u^2 \sigma_s^2} . \qquad (6)$$

This expression indicates that the uncertainty in profits is related solely to the speculative purchase or sale of forward contracts. Substituting (6) and the optimal choice for y_t in (5) into (3), the objective function can be rewritten as:

$$\max_{p_t} U\left((e_t p_t - E_{t-1}c_t^*)E_{t-1}x_t + ({}_{t-1}f_t - e_t)E_{t-1}x_t + \frac{({}_{t-1}f_t - e_t)^2}{R_u \sigma_s^2} \right)$$
$$+ \frac{1}{2} U''(E_{t-1}\pi_t^*) \frac{({}_{t-1}f_t - e_t)^2}{R_u^2 \sigma_s^2} . \qquad (7)$$

Treating $U''(E_{t-1}\pi_t^*)$ as fixed, the coefficient of absolute risk-aversion R_u will still vary with changes in p_t. However, working out the algebra shows that the derivatives of R_u with respect to p_t - in the two places where it appears in (7) - cancel each other out. Thus, the only terms that matter are the first terms that appear in (7), which are simplified as:

$$(e_t p_t - E_{t-1}c_t^*)E_{t-1}x_t + ({}_{t-1}f_t - e_t)p_t E_{t-1}x_t = ({}_{t-1}f_t p_t - E_{t-1}c_t^*)E_{t-1}x_t .$$

In other words, the firm will seek to maximise profits evaluated at the hypothetical own-currency price obtained at the forward exchange rate.

Letting $\eta_t \equiv -\partial \ln(E_{t-1}x_t)/\partial \ln p_t$ denote the elasticity of demand, the first-order condition for (7) is simply:

$$p_t\left(1 - \frac{1}{\eta_t}\right) = \left(\frac{E_{t-1}c_t^*}{{}_{t-1}f_t}\right). \qquad (8)$$

This is a conventional Lerner pricing formula, with the exporter's marginal costs $E_{t-1}c_t^*$ converted to the importing country's currency using the *forward rate* ${}_{t-1}f_t$. Thus, even when the revenue received from export sales is only partially covered by forward contracts, it is the forward rate that determines the optimal price. This is an illustration of the "separation theorem" discussed by Ethier (1973), Baron (1976a) and Eldor and Zilcha (1987). We next examine whether this same result holds with invoicing in the exporter's own currency.

2.2. Invoicing in the Exporting Country's Currency

The maximisation problem confronting an exporting firm which sets price in its own currency is similar to that above, except that now profit is maximised by choosing p_t^* and y_t:

$$\max_{p_t^*, y_t} U(E_{t-1}\pi_t^*) + \frac{1}{2} U''(E_{t-1}\pi_t^*) \mathrm{var}_{t-1}(\pi_t^*), \qquad (9)$$

where $\pi_t^* = (p_t^* - c_t^*)x(p_t^*/s_t, q_t, I_t) + y_t({}_{t-1}f_t - s_t)$ again denotes profits in the exporting country's currency, and p_t^*/s_t is the random price in the importing country. As before, the firm sets price before the exchange rate is known but, unlike the case where the exporter sets price in the domestic currency, revenues are uncertain due to random fluctuations in import price and demand. This means that the terms $E_{t-1}\pi_t^*$ and $\mathrm{var}_{t-1}(\pi_t^*)$ take on the form:

$$E_{t-1}\pi_t^* = (p_t^* - E_{t-1}c_t^*)E_{t-1}x_t + y_t({}_{t-1}f_t - e_t), \qquad (10a)$$

$$\mathrm{var}_{t-1}(\pi_t^*) = (p_t^* - E_{t-1}c_t^*)\mathrm{var}_{t-1}(x_t) + y_t^2\sigma_s^2 - 2y_t(p_t^* - E_{t-1}c_t^*)\mathrm{cov}_{t-1}(x_t, s_t). \qquad (10b)$$

Treating $U''(E_{t-1}\pi_t^*)$ as constant, the first-order condition for (9) with respect to y_t is:

$$y_t = \frac{({}_{t-1}f_t - e_t)}{R_u \sigma_s^2} + (p_t^* - E_{t-1}c_t^*)\left[\frac{cov_{t-1}(x_t,s_t)}{\sigma_s^2}\right]. \tag{11}$$

Thus, the desirability of forward covering depends on both the relation between the forward and expected future spot rate - which is the same speculative effect obtained in (5) - and on the covariance between the future spot rate and product demand. The latter term enters because changes in the exchange rate will affect the product price in the importer's currency, and therefore product demand, and the exporter will want to hedge against this "operating exposure." An unanticipated depreciation of the importer's currency lowers the spot rate s_t, which raises the price $p_t = p_t^*/s_t$, and lowers expected demand and profits. It follows that $cov_{t-1}(x_t,s_t)>0$, so the firm will hedge by *selling* forward contracts in the importer's currency. Then a depreciation of that currency results in greater profits earned on the forward contract, which offset the loss in profits on its reduced sales.

Substituting (11) back into (10b), we can rewrite the conditional variance of profits as:

$$var_{t-1}(\pi_t^*) = \frac{({}_{t-1}f_t - e_t)^2}{R_u^2 \sigma_s^2} + \frac{(p_t^* - E_{t-1}c_t^*)^2}{\sigma_s^2}|V_t| , \tag{12a}$$

where,

$$V_t = \begin{bmatrix} \sigma_s^2 & cov_{t-1}(x_t,s_t) \\ cov_{t-1}(x_t,s_t) & var_{t-1}(x_t) \end{bmatrix}. \tag{12b}$$

The first term on the right of (12a) reflects the uncertainly in profits due to the speculative purchase or sale of forward contracts. The second term depends on $|V_t|$, which reflects the correlation between the changes in the spot rate and product demand, since:

$$|V_t| = \sigma_s^2 var_{t-1}(x_t)\left(1 - \frac{cov_{t-1}(x_t,s_t)^2}{\sigma_s^2 var_{t-1}(x_t)}\right).$$

The magnitude of this term depends on the functional form of demand, as the following example makes clear.

Let the demand function be given by:

$$x(p_t, q_t, I_t) = \left(\frac{\alpha}{p_t} - \frac{\beta}{q_t}\right) I_t, \quad \alpha, \beta > 0 \ . \tag{13}$$

This functional form, while not familiar, has very conventional properties:[3]

(a) Decreasing function in own price, $x_p < 0$;

(b) An increase in the price of the domestic import-competing good, $x_q > 0$;

(c) If the price of imported good is sufficiently higher than domestic good, then demand for imported good will be zero: $x_t > 0$ only for $p_t < q_t(\alpha/\beta)$.

Substituting $p_t = p_t^*/s_t$ into (13), and keeping p_t^* fixed, it is immediate that changes in demand are *perfectly correlated* with changes in the spot rate s_t. In this case, the firm can entirely eliminate the uncertainty in its profits by selling forward contracts in the importer's currency. Formally, it is readily verified that for the demand function (13), $|V_t|=0$.

More generally, we would expect that for other functional forms of demand, the firm would still be able to eliminate the uncertainty in its profits arising from fluctuating price and demand if it had available a complete set of put and call options on the foreign currency. Then for any possible change in the spot rate, the exporter could calculate the corresponding change in expected demand and profits, and make the appropriate forward sale to offset the fluctuation in profits. In that case, the remaining variation in profits would consist of only the first term in (12a), reflecting the speculative holding of forward contracts. Thus, while we will focus on the special case of the demand function in (13), we expect that similar results would hold for more general demand functions and a complete set of exchange rate options.

Using $|V_t|=0$ in (12a), computing $cov_{t-1}(x_t, s_t)$ from (13) and using this in (11) and (10a), the objective function (7) can be rewritten as:

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[3] We adopt this functional form because it leads to a simple log-linear relationship between the exchange rate and the export price, as shown in section 3.1.

$$\max_{p_t^*} U\left((p_t^* - E_{t-1}c_t^*)\left(E_{t-1}x_t + ({}_{t-1}f_t - e_t)\frac{\alpha I_{t-1}}{p_t^*}\right) + \frac{({}_{t-1}f_t - e_t)^2}{R_u\sigma_s^2}\right)$$

$$+ \frac{1}{2} U''(E_{t-1}\pi_t^*) \frac{({}_{t-1}f_t - e_t)^2}{R_u^2\sigma_s^2} . \tag{14}$$

Again, the coefficient of absolute risk-aversion R_u will vary with changes in p_t^*, but the derivatives of R_u with respect to p_t^* in the two places where it appears in (14) cancel each other out. Thus, the only terms that matter are the first terms that appear in (14), which are simplified using (13) as:

$$(p_t^* - E_{t-1}c_t^*)[E_{t-1}x_t + ({}_{t-1}f_t - e_t)\alpha(I_{t-1}/p_t^*)]$$

$$= (p_t^* - E_{t-1}c_t^*)[\alpha({}_{t-1}f_t/p_t^*) - (\beta/q_t)]E_{t-1}I_t = (p_t^* - E_{t-1}c_t^*)[E_{t-1}x(p_t^*/{}_{t-1}f_t, q_t, I_t)].$$

In other words, the firm will seek to maximise profits evaluated at the hypothetical import price obtained at the forward exchange rate.

Letting $\eta_t^* \equiv -\partial \ln[E_{t-1}x(p_t^*/{}_{t-1}f_t, q_t, I_t)]/\partial \ln p_t^*$ denote the elasticity evaluated at this forward rate, the first-order condition for p_t^* is simply:

$$p_t^*\left(1 - \frac{1}{\eta_t^*}\right) = E_{t-1}c_t^* . \tag{15}$$

Notice the similarity with equation (8) for the exporter invoicing in the domestic currency. Again, a Lerner pricing formula is obtained, but with the forward exchange rate used in the elasticity to compute a hypothetical price in the importer's currency. This forward rate is used despite the fact that the firm may be only partially covering its operating exposure. Equation (8) and (15) show that with optimal forward covering, the variance of the spot rate *does not* affect the optimal price chosen by the exporting firm.[4]

[4] This result is not obtained if forward covering is ignored, as in Mann (1989), for example.

3. Empirical Model

3.1 Functional Form

In order to convert (8) and (15) into equations that can be estimated, it is very convenient to again use the demand function (13). This function implies a log-linear relationship between the chosen price and its determining variables. For the case where the exporter invoices in the importer's currency and its own currency, respectively, we obtain:

$$\ln p_t = \gamma_0 + \gamma_1(\ln E_{t-1}c^*_t - \ln_{t-1}f_t) + (1-\gamma_1)\ln q_t\,, \qquad (8')$$

and,
$$\ln p^*_t = \gamma_0 + \gamma_1 E_{t-1}\ln c^*_t + (1-\gamma_1)(\ln q_t + \ln_{t-1}f_t)\,, \qquad (15')$$

where $\gamma_0 = \frac{1}{2}\ln(\alpha/\beta)$ and $\gamma_1=1/2$. An appreciation of the exporter's currency in the forward market will lower ${}_{t-1}f_t$, raise the price of that product in the importer's currency in (8'), and lower the price received by the exporter in (15'). Both the log-linear form of these expressions, and the coefficients of $\gamma_1=1/2$, follow from the special form of demand in (11). For more general demand functions we can still obtain a log-linear form for these pass-through equations, but with other values for $\gamma_1 \neq 1/2$, as we shall allow. In general, the pass-through equations must be homogeneous of degree one in the right-hand side variables, so the coefficients sum to unity as shown above.[5]

Pass-through equations of the form (8') or (15') have been recently estimated on disaggregate data (e.g. Knetter, 1989, 1993; Feenstra, 1989; Marston, 1990), though using the spot rather than forward rate. The variables in these equations are sometimes found to be *cointegrated,* meaning that (some or all) of the variables are integrated of order one, I(1), but a linear combination is found to be stationary, or I(0). This will be the case for the equations estimated in this paper. Then without loss of generality, any variable with a nonzero coefficient can be treated as the "dependent" variable, with its coefficient normalised at unity. We will find it convenient to invert the pass-through equations to obtain a purchasing power parity (PPP) formulation.

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[5] The existence of demand function yielding log-linear pass-through equation is discussed in Feenstra (1989), where the homogeneity properties are also established.

Considering first the case where the exporter invoices in the importing country's currency, we can move the forward rate to the left of (8') to obtain:

$$\ln {}_{t-1}f_t = \left(\frac{\gamma_0}{\gamma_1}\right) + (E_{t-1}\ln c^*_t - \ln q_t) - \left(\frac{1}{\gamma_1}\right)(\ln p_t - \ln q_t) \ . \qquad (16)$$

This is interpreted as a PPP equation applied to the forward rate. The variable ($E_{t-1}\ln c^*_t - \ln q_{t-1}$) equals the exporter's costs relative to competing prices in the importing country, and its coefficient of unity accords with the conventional PPP equation: an increase in relative prices of the exporting country will depreciate its exchange rate, or raise ${}_{t-1}f_t$. The variable ($\ln p_t - \ln q_t$) is the import price relative to the domestic price, and whilst it does not normally appear in a PPP equation, our derivation from the optimal pricing behaviour of the exporting firm shows that this variable is relevant. The coefficient of this relative import price equals the inverse of the pass-through elasticity in (8').

To express (16) in terms of the sport rate, we can use the covered interest parity condition:

$$\ln {}_{t-1}f_t - \ln s_{t-1} = \ln[(1+i^*_{t-1})/(1+i_{t-1})] \approx (i^*_{t-1} - i_{t-1}) \ , \qquad (17)$$

where i^*_{t-1} (i_{t-1}) denotes the period t-1 nominal interest rate in the exporting (importing) country. Substituting this into (16), and combining variables, we obtain:

$$\ln s_{t-1} = \left(\frac{\gamma_0}{\gamma_1}\right) + (E_{t-1}\ln c^*_t - \ln q_t) - \left(\frac{1}{\gamma_1}\right)(\ln p_t - \ln q_t) + (i_{t-1} - i^*_{t-1}). \qquad (18)$$

Thus, the PPP equation for the spot exchange rate includes the interest rate differential as an explanatory variable, with a coefficient of unity.

To estimate (18), we need to determine the forecasted value $E_{t-1}\ln c^*_t$. We will assume that $\ln c^*_t$ is integrated of order one, I(1), and verify that this property holds empirically. This means that $E_{t-1}\ln c^*_t$ can be written as a linear combination of $\ln c^*_{t-1}$, $\ln c^*_{t-2}$,..., or alternatively, as a linear combination of $\ln c^*_{t-1}$ and $\Delta \ln c^*_{t-1}, \Delta \ln c^*_{t-2}, ...$[6] While the differences $\Delta \ln c^*_{t-1}$ and $\Delta \ln c^*_{t-2}$ are known to the firm at time t-1 and therefore nonstochastic, for the purpose of estimation they can be included in an error term of (18) and treated as stationary random variables. We will also assume that $\ln p_t$ and $\ln q_t$ in (18) are I(1), while verifying that this property holds empirically. This allows us to replace these variables by their lagged values in (18), and add other stationary errors onto the right. Then updating the subscript on all variables from t-1 to t yields:

$$\ln s_t = \left(\frac{\gamma_0}{\gamma_1}\right) + (\ln c^*_t - \ln q_t) - \left(\frac{1}{\gamma_1}\right)(\ln p_t - \ln q_t) + (i_t - i^*_t) + u_t \, . \qquad (18')$$

We expect that the spot exchange rate in also I(1), but the error u_t is stationary by construction; thus, (18') represents a cointegrating relation between the spot rate and the various prices. We will find that the interest differential is stationary, or nearly so, for most countries.

We next compare the results in (18) to those obtained when the exporting firm prices in its own currency. Moving the forward rate onto the left of (15'), and using the covered interest parity condition (17), we obtain:

$$\ln s_{t-1} = \frac{-\gamma_0}{(1-\gamma_1)} + (E_{t-1}\ln c^*_t - \ln q_t) + \frac{1}{(1-\gamma_1)}(\ln p^*_t - E_{t-1}\ln c^*_t) + (i_{t-1} - i^*_{t-1}). \qquad (19)$$

This equation differs from (18) in that the *exporter's price relative to marginal cost* appears on the right, rather than the relative import price. Otherwise, (18) and (19) are the same in that the variable $(E_{t-1}\ln c^*_t - \ln q_t)$, and the interest rate differential, still appear with coefficients of unity. We can write (19) in a stochastic form by replacing $E_{t-1}\ln c^*_t$ by $\ln c^*_{t-1}$ plus a stationary error, and similarly replacing $\ln p_t$ and $\ln q_t$ by their lagged values pus errors, to obtain:

[6] For example, if $\ln c^*_t$ follows that time series process $\ln c^*_t = \ln c^*_{t-1} + \varepsilon_t$ with $\varepsilon_t = \rho\varepsilon_{t-1} + \nu_t$, and ν_t uncorrelated over time, then $E_{t-1}(\ln c^*_t) = \ln c^*_{t-1} + E_{t-1}(\varepsilon_{t-1}) = \ln c^*_{t-1} + \rho(\ln c^*_{t-1} - \ln c^*_{t-2})$.

$$\ln s_t = \frac{-\gamma_0}{(1-\gamma_1)} + (\ln c_t^* - \ln q_t) + \frac{1}{(1-\gamma_1)}(\ln p_t^* - \ln c_t^*) + (i_t - i_t^*) + v_t \,. \qquad (19')$$

While (18') and (19') provide us with estimating equations when the exporter sets prices in the importer's currency, and its own currency, respectively, the price indices available in aggregate data would always be a combination of these two cases. To see how this affects the estimation, suppose that a fraction λ of products are priced in the importer's currency , using (18'), and the remaining $(1-\lambda)$ are priced in the exporter's currency using (19'). The import price index P_t is constructed as:

$$\ln P_t \equiv \lambda \ln p_t + (1-\lambda)(\ln p_t^* - \ln s_t), \qquad (20a)$$

where p_t is the nonstochastic price chosen by the exporter in the importing country's currency, whereas $(\ln p_t^* - \ln s_t)$ is the stochastic price of those imports whose price is set in the exporter's currency. Similarly, the export price index is constructed as:

$$\ln P_t^* \equiv \lambda(\ln p_t + \ln s_t) + (1-\lambda)\ln p_t^* \,, \qquad (20b)$$

where $(\ln p_t + \ln s_t)$ is the stochastic price received by the exporter on the fraction λ of products, while $\ln p_t^*$ is nonstochastic.

Summing $\lambda\gamma_1$ times (18') and $-(1-\gamma_1)(1-\lambda)$ times (19'), and using (20), the following relation between the spot rate, prices and interest differential is obtained:

$$\ln s_t = \frac{\gamma_0}{(\lambda+\gamma_1-1)} + (\ln c_t^* - \ln q_t) + (i_t - i_t^*)$$

$$- \frac{1}{(\lambda+\gamma_1-1)}\left[\lambda(\ln P_t - \ln q_t) + (1-\lambda)(\ln P_t^* - \ln c_t^*)\right] + \frac{1}{(\lambda+\gamma_1-1)} w_t \,, \qquad (21)$$

where $w_t = [\lambda\gamma_1 u_t - (1-\lambda)(1-\gamma_1)v_t]$, and $\lambda \neq (1-\gamma_1)$ is assumed. Thus, the spot rate depends on a *weighted average* of the relative import and export price indexes, which we shall refer to as the "relative traded goods price." The weights used reflect the proportion of traded goods prices in the importer's and exporter's currency, respectively. Note that this proportion λ also affects the coefficient of the average traded goods price. For $0 < \gamma_1 < 1$, an appreciation of the importer's currency (rise in s_t) will be associated with a fall in P_t and a rise in P^*_t, so that $(\lambda+\gamma_1-1)$ is negative or positive depending on whether most traded goods are priced in the importer's or exporter's currency. In particular, when $\lambda=1$ we obtain the coefficient on the relative import price in (18'), and when $\lambda=0$ we obtain the coefficient on the relative export price in (19').

3.2 Data and Identification

We will consider the PPP relation between the U.S. and four major trading partners - Canada, the Federal Republic of Germany, Japan and the United Kingdom (U.K.). Aggregate quarterly data from 1974:1 to 1994:4 comprise the sample period, most of which are taken from the International Monetary Fund, *International Financial Statistics* (IFS). Variables of equation (21) for the four countries are as follows. The dependent variable ($\ln s_t$) is the average quarterly spot rate of foreign currency per U.S. dollar. Period averages (IFS line rf) seem more reasonable than using end-of-quarter figures since exporters ship goods throughout the period. For the foreign marginal costs ($\ln c^*_t$) and the U.S. domestic price ($\ln q_t$) we use the wholesale price indices (WPI) of these countries (IFS line 63). The interest rate differential ($i_t - i^*_t$) is the difference between the 90-day interest rate for the U.S. and foreign countries (IFS line 60C, except 60L for Japan).

The export price index ($\ln P^*_t$) for each country except Canada is a *multilateral* price index, reflecting the trade of that country with the rest of the world. For the U.K. we use the export unit-value index (IFS line 74), but for Germany and Japan the export price indices (IFS line 76) are available. These indices contrast with our theoretical model, however, which is based on the *bilateral* price indices between the importing and exporting country. Since bilateral indices between the U.S. and its trading partners are not generally available, the multilateral indices are used instead. In contrast, for Canadian exports to the U.S. bilateral price indices for various sectors are available.[7] One defect of these indices is that those for "end products" are derived from Canadian *wholesale* price indexes. This procedure for constructing the export price indices invalidates their ability to measure the changing ratio of price to marginal cost (since marginal cost is also measured by the wholesale price index). To

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[7] These bilateral price indices are published by Statistics Canada, and were graciously provided by Lawrence Schembri of the Department of Economics, Carleton University.

avoid this, our measure of the Canadian export price index to the U.S. is taken from the "fabricated material, inedible" sector, which accounts for about one-third of the total value of exports to the U.S.[8]

For each country exporting to the U.S., the dollar value of the export price index was constructed by simply converting this index using the (period average) spot rate:

$$\ln P_t \equiv \ln P^*_t - \ln s_t , \tag{22}$$

which follows directly from (20). Thus, P_t measures the dollar value of the *multilateral export* price index for each country except Canada. Again, this contrasts with our theoretical model which depends on the *bilateral* price indexes.[9] This identity has implications for the identification of the coefficients in (21), as we shall discuss below.

Data on the currency of invoice for exports are available from Page (1981, Table 1) for the years 1979 or 1980, and Black (1991) for 1987. They list the same percentages of *total* country exports that are invoiced in the country's own currency: Germany, 82%; Japan, 33%; United Kingdom, 76%. These percentages correspond to $(1\text{-}\lambda)$ in our theoretical model, or the fraction of trade priced in the exporter's currency. For Canada, Page estimates that 15% of *total* exports are priced in Canadian dollars, but we expect that the fraction of exports to the U.S. priced in Canadian dollars is even smaller, and will use $(1\text{-}\lambda)=0$ in our initial calculations.[10]

With these values for $(1\text{-}\lambda)$, the average traded goods price $[\lambda(\ln P_t - \ln q_t)+(1-\lambda)(\ln P^*_t - \ln c^*_t)]$ that appears in (21) is computed. These values of $(1\text{-}\lambda)$ should be regarded a estimates, however, and it will be important to determine how sensitive our results are to other choices.[11]

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[8] The category of "end products, inedible," for which the wholesale price indices were used, accounts for 40-55% of Canadian exports to the U.S., while "crude materials, inedible" accounts for 10-15%. We did not consider price movements in the latter category because it was principally influenced by the price of oil. The remaining categories of exports are agricultural products, which account for about 5% of the value of trade.

[9] Another possibility would have been to measure P_t by the U.S. multilateral *import* price index, in which case (22) would not hold exactly. We experimented with this alternative measure, and found that the results were similar to those obtained with measuring the dollar value of export prices using (22).

[10] McKinnon (1979, pp. 73-74) reports that Canadian exports are seldom invoiced in Canadian dollars, drawing on the survey evidence from Boston firms in Fieleke (1971).

[11] For example, using *bilateral* data for a sample of products in 1973, Magee (1974, Table 2) finds that 28% of Japanese exports to the U.S. are invoiced in yen, which is the same figure that we calculate in (23). However, he also finds that between 60% or 81% (depending on the calculation) of German exports to the U.S. are invoiced in

To determine the sensitivity to the estimating equation to the percentages used for invoicing currencies, let λ continue to represent the true percentage invoiced in the importer's currency, as in (20). However, suppose that the relative traded goods price is constructed using the weight λ' as:

$$[\lambda'(\ln P_t - \ln q_t) + (1-\lambda')(\ln P^*_t - \ln c^*_t)] = \\ (\lambda - \lambda')(\ln s_t - \ln q_t - \ln c^*_t) + [\lambda(\ln P_t - \ln q_t) + (1-\lambda)(\ln P^*_t - \ln c^*_t)], \qquad (23)$$

where the equality follows from (20). Substituting (23) into (21), we obtain:

$$\ln s_t = \frac{\gamma_0}{(\lambda'+\gamma_1-1)} + (\ln c^*_t - \ln q_t) + \left(\frac{\lambda+\gamma_1-1}{\lambda'+\gamma_1-1}\right)(i_t - i^*_t) \\ - \frac{1}{(\lambda'+\gamma_1-1)}[\lambda'(\ln P_t - \ln q_t) + (1-\lambda')(\ln P^*_t - \ln c^*_t)] + \left(\frac{1}{\lambda'+\gamma_1-1}\right) w_t . \qquad (24)$$

Comparing (21) and (24) it is evident that mispecification of the weight λ - using λ' instead - will affect the coefficient of the interest rate differential and the relative traded goods price, though the coefficient of the relative wholesale prices $(\ln c^*_t - \ln q_t)$ is not affected. Another way of stating this result is that the coefficients of (21) are not identified without knowledge of λ, so that we could not estimate this parameter along with the other coefficients. The reason for this identification problem is that the ratio of export and import prices is exactly equal to the spot rate, or $\ln s_t \equiv \ln P^*_t - \ln P_t$ as in (22). Adding any multiple of this identity to (21), we obtain other linear combinations of the variables that would have the same residuals (up to scalar multiple), and in this sense, explain the data equally well. We will find that the likelihood value for our estimated model *does not depend* on the value of λ, though the coefficients of the interest rate differential and relative traded goods price are affected. After reporting results for the values for λ discussed above, we will also determine the sensitivity of the coefficients to other values.

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marks, which is higher than in (23). Grassman (1973,1976) provides evidence on the currency of invoice for exports from Sweden and Denmark.

The identity (22) also has implications for testing the hypothesis that wholesale prices are mismeasured due to the inclusion of nontraded goods. Rewriting (22), we obtain;

$$\ln s_t \equiv (\ln c^*_t - \ln q_t) - (\ln P_t - \ln q_t) + (\ln P^*_t - \ln c^*_t) \ . \qquad (22')$$

This can be interpreted as saying that the PPP equation defined over wholesale prices should be corrected by including the relative import and export prices, in which case PPP holds by construction when the bilateral price indices are used. Note that there is an important difference between (22') and our specification (21) or (24), in that the relative traded goods prices appear with *opposite sign* in (22'), but with the *same sign* in (21) or (24), for $0 \leq \lambda \leq 1$. This means that the when estimating (21) or (24), there is no possibility of simply obtaining the identity (22'). Evidently, if one wanted to test whether deviations from PPP were caused by the mismeasurement of wholesale prices (due to the inclusion of nontraded goods), a hypothesis that does not simply yield an identity like (22') would need to be developed.

4. Estimation

4.1. Testing for Unit Roots

The first task in estimation is to determine whether the variables are stationary or not. We use the augmented Dickey-Fuller (ADF) test, under which the null hypothesis is that the variable has a unit root (Dickey and Fuller, 1991). This hypothesis is tested by regressing the difference of a variable on a constant, its own lagged value, lagged differences, and possibly a time trend.[12] Under the null hypothesis, the coefficient of the lagged value should be insignificantly different from zero. The ADF test statistic is just the ratio of the coefficient to its standard error (though this does not have a t-distribution). These test statistics are reported in Table 1, where the first row for each variable tests the null hypothesis of a unit root in the *level*, and the second row tests the null hypothesis of a unit root in the *first-difference*. We

[12] We included three quarterly lags of the differences, which were sufficient to eliminate serial correlation in the errors, except for the Canada WPI where four lags were used.

also included a time trend in the test when its coefficient was significant at the 10% level, as indicated by "t" in the table.

Looking first at the results for the spot exchange rates (relative to the dollar), we cannot reject the null hypothesis of a unit root in the levels, but do reject the hypothesis of a unit root in the first-differences. As expected, we conclude that these spot rates are I(1). The wholesale price indices are also found to be I(1) for Germany and Japan. For Canada, we cannot reject the hypothesis of a unit root in either the levels or first-differences so it appears that this variable is I(2). The economic interpretation is that the *inflation rate* for Canada appears to have a unit root, which is surprising.[13] For the U.K., we find that the wholesale price index is stationary.

Turning to the relative traded goods price, $[\lambda(\ln P_t - \ln q_t) + (1-\lambda)(\ln P^*_t - \ln c^*_t)]$, we conclude that this variable has a unit root for all countries except Japan, in which case we find that the variable is stationary at the 10% significance level. The interest rate differential is found to be stationary for Canada and the U.K., and possibly also for Japan at a weaker significance level. The interest rate differential between Germany and the U.S. stands out as failing to reject a unit root in the levels, but soundly rejecting a unit root in the first differences, so that it is I(1). Summing up, most variables in (21) are found to have a unit-root, except for the interest differential which is stationary for most countries, and the WPI which can be integrated of various orders.

4.2. Estimation of Cointegrating Relations

The parameters of (21) are estimated using the method of Johansen (1991), which imbeds this cointegrating relation within a vector-autoregression (VAR) system of all five variables, denoted by the column vector $X_t \equiv [\ln s_t, \ln c^*_t, \ln q_t, (i_t - i^*_t), \lambda(\ln P_t - \ln q_t) + (1-\lambda)(\ln P^*_t - \ln c^*_t)]'$. The VAR specifies that the difference of each variable depends on a constant,

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[13] The hypothesis that the second-difference has a unit root is strongly rejected. Using monthly data, Pippenger (1993) also finds evidence that the WPI of some countries are I(2). For the U.S. WPI (not shown in Table 1), the ADF test statistic is -1.60 for the level, and -2.32 for the first-difference. Thus, we also conclude this variable is I(2), though it might be I(1) at a significance level weaker than 10%. For both the U.S. and Canada, these results are sensitive to the sample period that is used, and omitting the last three years leads one to conclude that the WPI for both countries are stationary.

lagged differences, and lagged cointegration relations that are linear combinations of the five variables:

$$\Delta X_t = \mu + \sum_{k=1}^{K} \theta_k \Delta X_{t-k} + \sum_{i=1}^{N} \alpha_i \beta_i' X_{t-1} + \varepsilon_t \,, \qquad (25)$$

where μ is a (5x1) vector of constants, K is the number of lagged differences, θ_k is a (5x5) matrix of estimated coefficients for each lag k, N in the number of cointegrating relations, and α_i and β_i are (5x1) vectors of estimated coefficients. In particular, β_i is the cointegrating vector such that the cointegrating relation $\beta_i' X_{t-1}$ is stationary. The cointegrating relation can be interpreted as an error-correction term, which adjusts the change in each variable in the VAR according to the error from the long-run equilibrium. It is quite possible that there are *multiple* cointegrating relations, which we have indexed by i. Johansen (1991) derives the maximum likelihood estimates of coefficients in (25), and shows how to test for the number of cointegrating vectors β_i. In comparison with single-equation method for estimating cointegrating vectors, such as Engel and Granger (1987), the Johansen method is thought to be more powerful by using the full VAR system, and also has the advantage that the standard errors of the cointegrating vector(s) are normally distributed.

We first check for the number of cointegrating vectors. With five variables in our system, the maximum number of cointegrating vectors obtained from the estimation is also five. By construction, these vectors are independent. Since any linear combination of the cointegrating relations is also stationary, finding five such relations would mean that all of the variables in the system are stationary. We have already found that this is not the case for the unit root tests. A reduction in the number of cointegrating vectors is tested by a likelihood ratio test, where the null hypothesis is that the number is at most N. In Table 2 we report the results of these likelihood ratio tests for each value of N.[14]

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[14] All empirical results in this section were computed on Econometric Views version 1.0, which also provides the critical values for Table 1. For most countries, three quarterly lags of the differenced variables were included, along with a constant term in each cointegrating relation and in each equation of the VAR system. The exception is Canada, for which four quarterly lags were used, and a time-trend in the cointegrating relations was also permitted because it was highly significant. Because of the differing treatment of Canada, significance levels higher than those shown in Table 2 apply.

For Canada, the likelihood ratio tests indicate that we cannot reject the hypothesis that the number of cointegrating vector is at most two at the 5% critical level, though it is likely that at the 10% level we would conclude there are three vectors. This number of stationary relations is found for Japan. For Germany and the U.K. we find evidence of *five* cointegrating vectors at the 5% level, though this contradicts our finding from Table 1 that the interest rate differential for Germany is nonstationary. Minor changes in the VAR for the U.K. results in the conclusions that there are four stationary relations.[15] While there is obviously some difference across the countries, we will proceed by treating the number of cointegrating relations in each case as at least three.

Johansen and Juselius (1990) suggest that the first cointegrating vector - which is associated with a certain maximum eigenvalue - is of special significance in that it is the "most correlated with the stationary part of the model" (p. 192). For all five countries, we find that the first cointegrating vector provides quite reasonable estimates of equation (21). In the first row for each country in Table 3 we report this cointegrating vector, where we have normalised the coefficient of the spot rate at unity, and expressed the other coefficients as appearing on the right-hand side of (21).

The spot rates are measured as each country's currency per U.S. dollar, so the expected coefficients on the wholesale prices are unity for the country's own WPI, and negative one on the U.S. WPI. The expected signs are obtained in most cases, but the magnitudes of some coefficients differ considerably from unity. The coefficient of the relative traded goods price can be of either sign, and is highly significant for all countries. The interest rate differential is measured as the U.S. relative to the foreign country's rate, and has a predicted coefficient of unity. A point estimate within one standard error of this magnitude is obtained for the U.K., while larger and highly significant estimates are obtained for Germany and Japan, but for Canada the estimate obtained is insignificantly different from zero. Thus, the relative trade

[15] The system (25) was estimated both with and without constant terms in each equation of the VAR, and the Schwartz criterion indicated that the constant was generally needed. For the U.K., the Schwartz criterion was ambiguous, and if the constant term is instead excluded then the number of cointegrating vectors is reduced to four. This occurs because of a change in the critical values for the likelihood ratio test, with minimal change to the cointegration estimates.

goods price appears to be an important determinant of the exchange rates in the PPP equation, and the interest rate differential is also significant in most cases.[16]

For all countries except Germany, the second unnormalised cointegrating vector (not reported in Table 3) has its largest coefficient on the interest rate differential, and smallest coefficient on the spot exchange rate. This result is not surprising since, for most countries, the results in Table 1 showed that interest rate differential was stationary, or nearly so. For the purposes of this paper, this second cointegrating vector is not of special interest in itself, but by taking a linear combination of the first and second vectors we can solve for a cointegrating relation with a zero coefficient on the interest rate differential. This vector is shown in the second row for each country (except Germany), again normalised as in (21).

If we continue with this method, a third cointegrating relation can be obtained by using a linear combination of the three estimated vectors, to obtain zero coefficients on the interest rate differential and the relative traded goods price. This results is a stationary relation between the spot rate and the foreign and U.S. WPI. Cheung and Lai (1993) have found that a stationary relation of this type holds for a number of countries, and we obtain reasonable estimates for Canada, Japan, and the U.K., which are reported in the third row for these countries.

For Germany, the same exercise of eliminating the interest rate differential leads to nonsensical results: the cointegrating vector obtained has coefficients of about 13 on the German WPI and -8 on the U.S. WPI, and even more extreme coefficients are obtained if the relative traded goods price is also excluded. If these coefficients are used to measure the residual in equation (21), then the deviations from PPP are greater than the fluctuations in the mark/dollar rate itself. Thus, while the cointegrating relation obtained by eliminating the interest rate differential is in principle stationary, the results obtained in this case are not meaningful.

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[16] The coefficient of the interest rate differential is quite sensitive to alternative specifications of the VAR system (25), such as changing the numbers of lags or inclusion of a time trend, whereas the coefficient of the relative traded goods price is not sensitive to these changes.

To obtain a second cointegrating vector for Germany, we make use of the fact that the coefficients on the German and U.S. WPI in Table 3 are quite similar, but with opposite signs. Thus, we consider imposing the "symmetry" restriction that these variables have the same coefficient (with opposite sign) in the PPP equation. We use a linear combination of the first two estimated vectors to obtain such a cointegrating relation, which is reported in the second row for Germany. We see that the resulting coefficient of the WPI is 1.80 (with a standard error of 0.19) which is closer to its expected value of unity than that obtained in the first cointegrating vector. For comparison, in the third row for Germany we report an OLS regression of the spot rate on the wholesale prices, which is used below.

Summing up, the first row of estimates in Table 5 is the first cointegrating vector, while the second two is obtained as linear combinations of the two estimated vectors, and similarly for the third row (except for Germany).[17] The developed theory on cointegration offers little guidance on how to interpret multiple vectors. The particular linear combinations we have chosen appear to be meaningful in terms of the economic interpretation, and the coefficient values obtained. The presence of multiple cointegrating vectors means that there are several combinations of variables, including the traditional PPP formulation, that result in stationary relations. It is noteworthy that the estimates in each cointegrating vector are consistent, and can be viewed as long-run equilibrium relations in the data.

To obtain additional insight into the cointegrating relations, we substitute the coefficients from Table 3 into (21) to calculate the residuals, which measure the deviations from PPP. These deviations are shown in Figures 1-4. In each Figure, the bold line shows the deviations from PPP as measured using only the data on the spot rate and wholesale prices. Thus, for Canada, Japan, and the U.K. the bold line - labeled "PPP3" - is the residual from the third cointegrating vector reported in Table 3. In contrast, for Germany we did not report a cointegrating vector between just the spot rate and wholesale prices, and instead use the residual from an OLS regression between these variables (also shown in Table 3) to calculate

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[17] The standard errors reported in Table 3 are provided by Econometric Views, but should be interpreted with caution. In particular, the standard errors on the Nth cointegrating vector in Table 3 (N=1,2,3) are computed under the hypothesis that there are only N cointegrating vectors in the system, whereas the correct standard errors would be computed under the maintained assumption that there are 3 cointegrating vectors.

the bold lines labeled “PPP3.” In Table 3 we also report the standard deviations of these calculated residuals.

The dashed lines labeled PPPi in each figure are the residuals calculated from the i'th cointegrating vector reported in Table 3, i=1,2. In all cases, these dashed lines show less variation than the bold lines, and have lower standard deviations, as reported in Table 3. In addition, the dashed lines are quite close, except for Germany and to a lesser extent Japan. The similarity of PPP1 and PPP2 indicates that the interest rate differential is quantitatively not important in explaining deviations from PPP, despite its significance in the estimated PPP relation. For all countries other than Germany, it is the relative traded goods price that is primarily responsible for explaining the deviations from PPP. By comparing the standard deviation of PPP3 with that of PPP1 or PPP2 in Table 3, we see that about one-sixth of the deviations from PPP is explained by the relative traded goods price for Canada, and more than one-third for Japan and the U.K.

It is also possible to compare the estimated coefficients on the relative traded goods price in Table 3 with their theoretical values. From (21), the sign of the estimated coefficient is the same as $(1-\lambda-\gamma_1)$, where γ_1 is the pass-through coefficient that is expected to lie within the range (0,1). For intermediate values of γ_1, it follows that the coefficient on the relative traded goods price will be positive (negative) when the proportion of the country’s exports denominated in dollars, λ, is low (high). For Canada and Japan, we have used a value for λ close to unity, so we expect a negative coefficient on the relative traded goods price, as was obtained in Table 3. The explanation for this sign is that an appreciation of the U.S. dollar will lead to a *fall* in the dollar price of these country’s exports, so there is a negative correlation between relative traded goods price and the exchange rate.

Conversely, for Germany and the U.K. we have used values for λ that are close to zero. In that case an appreciation of the dollar is expected to lead to a *rise* in the price of the export denominated in its own currency, as the foreign firms expand their price-cost margins. Consistent with this explanation, we find a positive cofficient on the relative traded goods price for the U.K. in Table 3, but *not* for Germany. To explain this puzzling result, we note that the expected positive correlation between the value of the dollar, and the export price relative to

WPI abroad, is evident from a graph of these variables for the U.K. (and also for Canada and Japan), but not for Germany. In other words, even in our raw data Germany does not display the kind of pass-through behaviour between export prices and exchange rates that we normally expect. Taken together with the unusually strong contribution of the interest rate differential to the PPP equation, we regard the overall results for Germany as anomalous.

4.3. Sensitivity of Results

In section 3.2, we argued that there was a problem in identifying the parameter λ, which we avoided by using data on the currency of invoice for exports from the various countries. The source of this identification problem was the identity (22), stating that the ratio of the bilateral price indices equals the spot exchange rate. Summing any multiple of this identity and the estimating equation (21), we obtain an alternative estimating equation with different coefficient values, but the same residuals (up to a scalar multiple). Taking linear combination of variables in this manner has *no effect* on the maximum likelihood value for the system (25), so it is impossible to estimate the value for λ.

On the other hand, experimenting with different values will affect the coefficients on the interest rate differential and relative traded goods price, as was demonstrated by (24). In the first two rows for each country in Table 4, we report the results from estimating the cointegrating vectors with alternative values of λ. Only the first cointegrating vector is reported, along with the standard deviation of the residuals from that vector. In some cases, the alternative values for λ move the estimated coefficient on the interest rate differential closer to its expected value of unity. The standard deviation of the residuals are also quite sensitive to λ (compare the first two rows of Table 4 with the first row of Table 3, for each country).

In the third row of Table 4 for each country, we report another sensitivity exercise, where we include the U.S. export price index relative to the U.S. WPI as new variable. The motivation for this exercise is that all the arguments we have made about pass-through behaviour apply equally well to U.S. exporters, so that their relative export price should also enter into the PPP equation. We have chosen to measure these exports as denominated entirely in U.S. dollars, as suggested by the evidence in Page (1981) and Black (1991). The relative export price enters as a significant variable for all the countries in Table 4, but in no case is the standard deviation of the PPP residuals less than that in the first row for each country in Table 3. From this very limited evidence, the U.S. relative export price does not appear to contribute extra information towards explaining the PPP residuals.

5. Conclusions

Froot and Rogoff (1995) have recently surveyed the evidence on PPP, and considered various explanations, including "pricing to market" or pass-through behaviour, to explain its failure. They conclude that "Pricing to market is an interesting and important issue. Because, however, it fundamentally derives from short-term rigidities, it seems unlikely to explain the medium and long-term deviations from PPP that we have been focusing on here" (p. 43). In contrast to this finding, we have found that pass-through behaviour appears to explain a significant portion of the deviations from PPP observed during the floating rate period since 1974. The variable we have used to measure pass-through behaviour is a weighted average of import relative to domestic prices, and export prices relative to costs of production. Either of these relative traded goods prices are often treated as the dependent variable in conventional pass-through equations, and by inverting these equations, we obtain a PPP formulation where the weighted average appears (along with wholesale or consumer prices) as a determinant of exchange rates. We have found this weighted average is significantly correlated with exchange rate movements, and in some cases, can explain one-third or more of the observed deviations from PPP.

We have also found that the interest rate differential enters the PPP equation as a significant variable, though it does not generally explain a noticeable portion of the deviations from PPP. One reason for this is that the interest rate differentials are themselves small in magnitude, and have greater stationarity, than other variables in the PPP relation. Johansen and Juselius (1992) have explored the PPP and uncovered interest parity relations in a cointegration setting similar to ours, and they also find evidence that the interest rate differentials enter as significant variables. Focusing on the United Kingdom, they use the *multilateral* exchange rate weighted by the trading partners of the U.K., along with the trade-weighted foreign WPI's, and the domestic and Eurodollar interest rates. This approach neatly resolves the non-conformity in our data between using the bilateral exchange rates, and the multilateral export price indexes: instead, the multilateral variables could be used throughout. This is one direction for further research.

Another direction for research is to imbed our modified PPP equation into a model of exchange rate determination, such as the monetary model of exchange rate determination presented by Woo (1985) and West (1987). West (1987) emphasises that *stochastic deviations* from PPP (and shocks to money demand) play a crucial role in his failure to reject the monetary model, and states that “It is therefore of interest in future work to model these shocks as functions at least in part of observable economic variables” (p. 72). Our PPP equation, obtained from the pass-through behaviour of optimising firms, can be considered a first step towards this goal.

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Table 1

Unit Root Tests: Augmented Dickey-Fuller Statistic

Variable	Canada	Germany	Japan	U.K.
Spot exchange rate				
Level	-2.26[t]	-2.26[t]	-2.80[t]	-2.25
First-difference	-3.64[a]	-3.89[a]	-3.94[a]	-4.08[a]
Wholesale price index				
Level	-1.17	-1.39	-2.03	-3.52[t,b]
First-difference	-2.17	-2.54[c]	-3.27[b]	-2.86[b]
Rel. traded goods price				
Level	-2.99[t]	-2.07	-2.66[c]	-2.68[t]
First-difference	-2.93[b]	-2.82[c]	-3.55[a]	-5.10[a]
Interest rate differential				
Level	-3.32[b]	-2.28[t]	-2.16	-2.81[c]
First-difference	-5.14[a]	-4.24[a]	-4.85[a]	-6.18[a]

Notes

All series are quarterly from 1974:1 to 1994:1, excluding 1974:1-1975:2 for the German interest rate differential. The ADF test was run with 3 quarterly lags of the differences, and a time trend was included when it was significant.

[a] Rejection of the null hypothesis of nonstationarity at the 1% level, where the critical value is -3.51 with no time trend and -4.07 with a time trend.

[a] Rejection of the null hypothesis of nonstationarity at the 5% level, where the critical value is -2.90 with no time trend and -3.47 with a time trend.

[a] Rejection of the null hypothesis of nonstationarity at the 10% level, where the critical value is -2.58 with no time trend and -3.25 with a time trend.

[t] Time trend was significant at the 10% level, and was included in the ADF test.

Table 2

Likelihood Ratio Tests for the Number of Cointegrating Vectors

Hypothesised Number	**5% Critical Value**	**Canada**[a]	**Germany**	**Japan**	**U.K.**
None	68.5	118.3[b]	90.8[b]	97.3[b]	125.8[b]
≤ 1	47.2	70.3[b]	56.0[b]	61.7[b]	70.5[b]
≤ 2	29.7	38.8	32.6[c]	35.5[c]	49.5[b]
≤ 3	15.4	21.7	17.7 [c]	12.6	20.3[b]
≤ 4	3.8	8.7	5.4 [c]	2.4	5.3 [c]

Notes

[a] Higher critical values apply for Canada, because a time-trend is included.

[b] Significant at the 1 percent level.

[c] Significant at the 5 percent level.

<u>Table 3</u> - Cointegrating Relations

Country	Interest Differ-ential	Relative Traded Goods Price	Country WPI	US WPI	Standard Dev. of Residual	Log Likeli-hood
Canada (λ=1)	0.55 (0.32)	-0.75 (0.090)	1.31 (0.081)	-1.17 (0.10)	0.039	1101.8
		-0.77 (0.090)	1.35 (0.082)	-1.19 (0.10)	0.040	
			0.92 (0.29)	-0.55 (0.36)	0.047	
Germany (λ=0.18)	3.16 (0.31)	-2.50 (0.22)	4.25 (0.41)	-3.14 (0.26)	0.069	1047.3
	4.03 (1.07)	-1.59 (0.87)	1.80 (0.19)	-1.80 (0.19)	0.112	
OLS			2.89 (0.60)	-2.21 (0.38)	0.158	
Japan (λ=0.77)	2.69 (0.62)	-1.72 (0.22)	0.72 (0.27)	-1.66 (0.037)	0.063	1014.2
		-1.20 (0.18)	1.54 (0.17)	-1.74 (0.032)	0.049	
			2.35 (0.19)	-1.90 (0.077)	0.103	
United Kingdom (λ=0.24)	1.76 (0.92)	3.09 (0.57)	1.04 (0.29)	-1.38 (0.41)	0.200	899.3
		3.03 (0.65)	0.94 (0.31)	-1.74 (0.53)	0.265	
			-1.04 (0.48)	1.04 (0.96)	0.346	

<u>Notes</u>:

The sample period for all countries is 1974:1-1994:4, excluding 1974:1-1975:2 for Germany.

All cointegrating regressions include a constant term, and a time trend is also used for Canada.

Table 4 - Sensitivity of Cointegrating Relations

Country	λ	Interest Differential	Relative Traded Goods Price	Country WPI	U.S. WPI	U.S. Relative Export Price	Standard Dev. of Residual
Canada	0.85	0.62 (0.35)	-0.85 (0.12)	1.35 (0.091)	-1.19 (0.11)		0.044
	0.70	0.71 (0.40)	-0.97 (0.15)	1.40 (0.10)	-1.22 (0.13)		0.051
	1.00	0.52 (0.35)	-0.91 (0.13)	1.23 (0.10)	-1.01 (0.13)	1.32 (0.52)	0.046
Germany	0.10	3.95 (0.33)	-3.12 (0.34)	5.06 (0.50)	-3.68 (0.29)		0.087
	0.30	2.43 (0.28)	-1.92 (0.13)	3.50 (0.37)	-2.65 (0.22)		0.053
	0.18	4.48 (0.59)	-1.33 (0.45)	4.98 (0.60)	-3.62 (0.37)	-2.31 (0.72)	0.085
Japan	0.70	3.05 (0.72)	-1.95 (0.28)	0.68 (0.31)	-1.75 (0.039)		0.072
	0.90	2.20 (0.48)	-1.40 (0.14)	0.77 (0.21)	-1.54 (0.037)		0.052
	0.77	-1.13 (0.58)	-2.16 (0.45)	1.10 (0.23)	-1.58 (0.038)	2.78 (0.63)	0.081
United Kingdom	0.10	1.22 (0.66)	2.16 (0.28)	1.03 (0.20)	-1.27 (0.27)		0.140
	0.40	3.48 (1.80)	6.11 (2.24)	1.07 (0.58)	-1.76 (0.89)		0.396
	0.24	1.16 (0.89)	5.32 (0.78)	1.66 (0.30)	-2.00 (0.40)	8.02 (1.55)	0.288

Notes:
The sample period for all countries is 1974:1-1994:4, excluding 1974:1-1975:2 for Germany.
All cointegrating regressions include a constant term, and a time trend is also used for Canada.

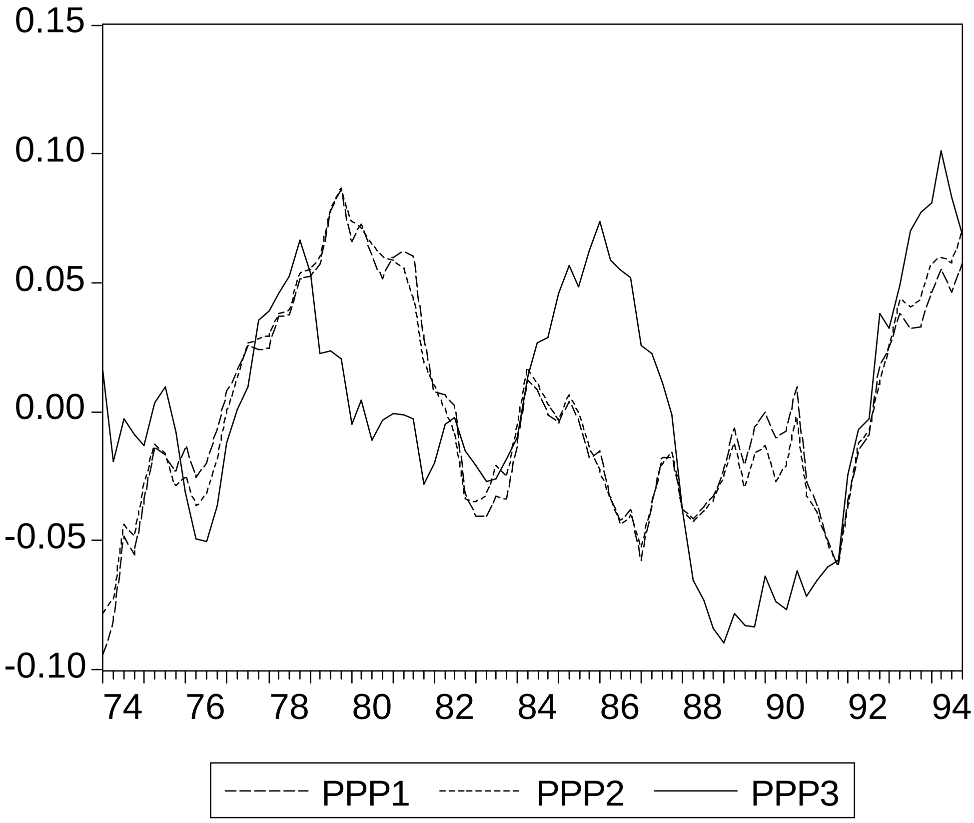

Figure 1: Cointegration Residuals for Canada

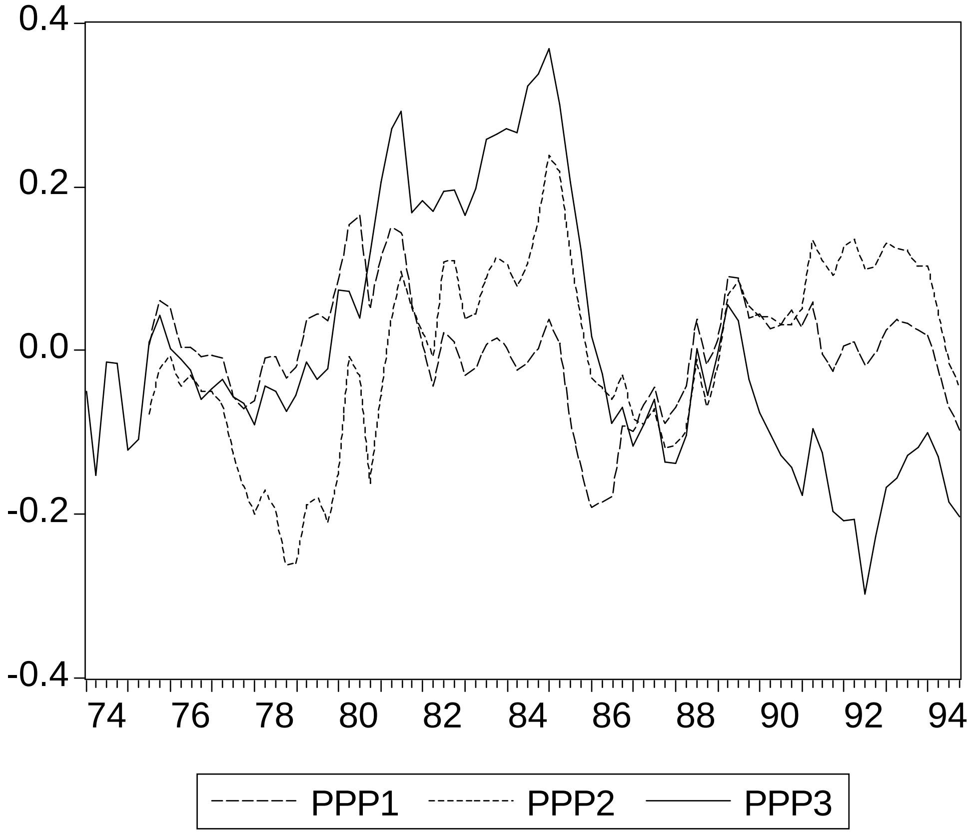

Figure 2: Cointegration Residuals for Germany

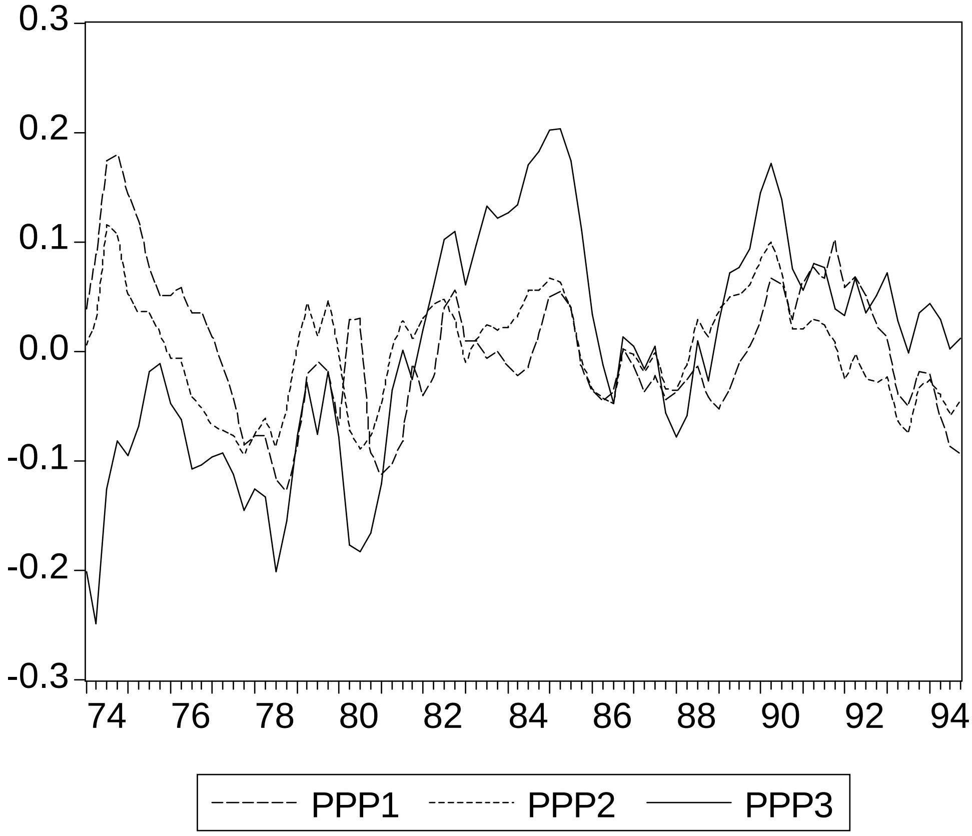

Figure 3: Cointegration Residuals for Japan

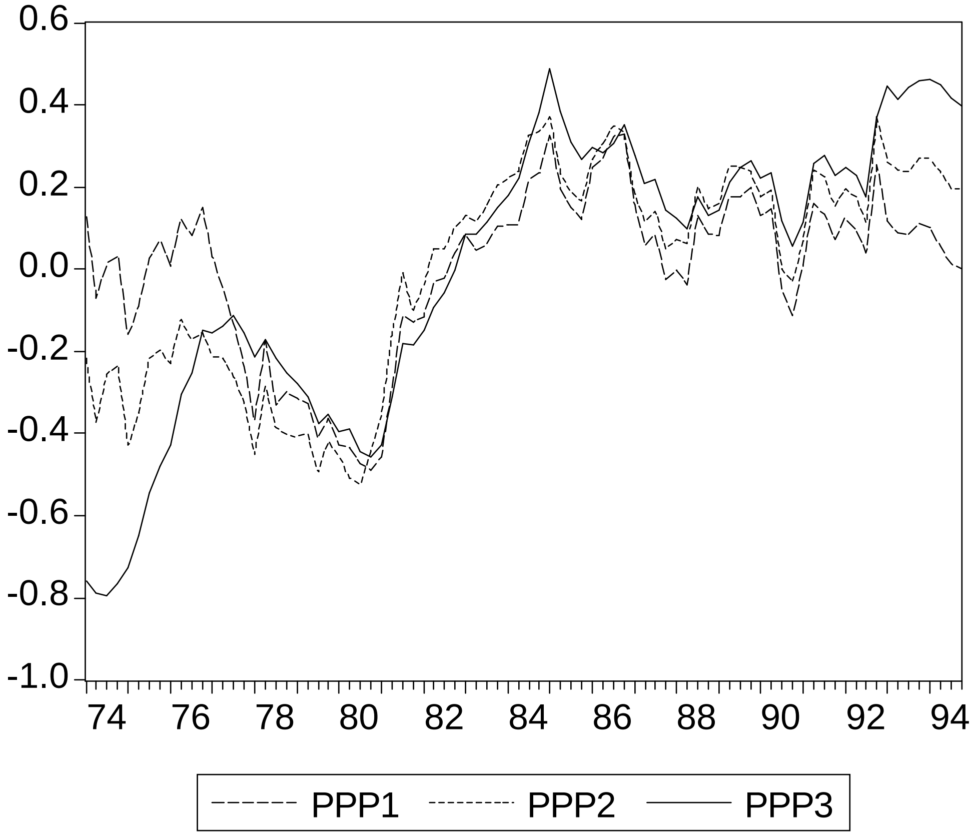

Figure 4: Cointegration Residuals for the U.K.